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DEVELOPMENTS IN DISCRETE
DISTRIBUTIONS 1969-1980

by

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Abstract

In this paper we survey and summarize developments in theory and methodology of discrete distributions during the period 1969-1980 since publication of our book Distributions in Statistics - Discrete Distributions (Wiley, 1969). A comprehensive (though not exhaustive) bibliography of some 400 items is included.

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DEVELOPMENTS IN DISCRETE DISTRIBUTIONS 1969-1980

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1. General Remarks

The bibliography of this article is intended to include all new results (of which we are aware) relating to discrete distributions which have appeared since publication of our book (Johnson and Kotz (1969) - referred to as JK), up to mid-1980. There are also included some earlier references which were not (although they should have been) included in JK.

The relevant literature contains a large number of specific results, some of theoretical and/or practical importance, but some relatively less important, though sometimes of aesthetic interest. In this text we try to describe a substantial part of the former but severe selectivity has been inevitable. In particular we have included little on approximations, characterizations and methods of estimation. We will first discuss some general methods, applicable to almost all distributions and then describe studies on broad classes or systems of distributions - some already well-known in 1969, and some not so well-known then, or newly introduced subsequent to that year.

We believe that formulation of appropriate broad classes of distributions can enhance understanding of the armory of available distributions, and the relationships among them. This can be of much assistance in the construction of models, and in the derivation of statistical techniques by analogy with known

techniques for closely related distributions.

The qualification "appropriate" in the previous paragraph is important. We do not regard the mere number and variety of distributions included in a class (or system) as being, in themselves measures of its importance - still less of its practical value. (For example the class for which $\sum_x \Pr[X=x] = 1$ is very broad, but of no value for our purposes.) What is important, and useful, is inclusion of as *wide a variety* of distributions as possible, within as *specific a formulation* as possible. This removes the need for piecemeal derivations and represents considerable economy of effort.

2. Notation

Unless otherwise indicated, we will *always* assume that the distributions we discuss here are lattice distributions over nonnegative values of the variable ($x = 0,1,2\ldots$).

We will use the abbreviations:

P_x for $\Pr[X=x]$; $P_{\underline{x}}$ for $\Pr[\bigcap_{j=1}^{m}(X_j=x_j)]$;

cdf for "cumulative distribution function";

pdf for "probability density function" (of a *continuous* variable);

pgf for "probability generating function" ($E[s^X] = \sum_x P_x s^x$);

$\sim$ for "is distributed as".

The distribution obtained by ascribing a distribution F_2 to parameter(s) $\underline{\theta}$ of a distribution F_1 will be denoted

$$F_1 \underset{\underline{\theta}}{\wedge} F_2 . \tag{1}$$

This is called a *compound* F_1 distribution; F_2 is the *compounding* distribution.

The distribution (1) is often called a "F_2-F_1" distribution (though sometimes, and in our opinion preferably, the order is reversed).

Denoting the pgf corresponding to a distribution F_1 by $G_1(s)$ the distribution with pgf obtained by replacing s in $G_1(s)$ by $G_2(s)$ - (giving $G_1(G_2(s))$) is termed the F_2-generalized F_1 distribution, and denoted

$$F_1 \vee F_2 \tag{2}$$

The distribution F_2 is termed the *generalizing* distribution. (Note that while F_2 in (1) may be continuous, F_2 in (2) must be a discrete lattice distribution.) A simple interpretation of (2) is obtained by noting that $\{G_2(s)\}^N$ is the pgf of the sum (convolution) of N independent F_2 variables, so that (2) is the distribution of a "random sum" of N such variables, the number of variables in the sum itself having a F_1 distribution. Boswell and Patil (1970) and Chatfield and Theobald (1973) have suggested the name *random sum* distribution for such distributions, in preference to "F_2-generalized", as the latter term may be confused with other generalizations of a different nature. Douglas (1970) refers to these distributions as randomly stopped sums. Chatfield and Theobald (1973) also suggest the use of the term "mixture distributions" in place of "compound distributions". Regarding N as a parameter of the N^{th} convolution, F_1^{*N} say, of F_1, (2) is also the compound distribution

$$F_1^{*N} \underset{N}{\wedge} F_2 \tag{3}$$

We will use some fairly standard notations for distributions such as

Poisson (θ)	$[P_x = e^{-\theta}\theta^x/x!]$	($\theta>0$; $x=0,1,\ldots$)
Binomial (n,p)	$[P_x = \binom{n}{x}p^x(1-p)^{n-x}]$	($0<p<1$); $x=0,1,\ldots,n$)
Negative Binomial (N,P)	$[P_x = \binom{N+x-1}{x}Q^{-N-x}P^x]$	($Q-1=P>0$; $x=0,1,\ldots$)
Logseries (θ)	$[P_x = (-\log(1-\theta))^{-1}\theta^x/x]$	($x=1,2,\ldots$)
Hypg (n,D,N)	$[P_x = \binom{D}{x}\binom{N-D}{n-x}/\binom{N}{n}]$	($\max(0,n-N+D)\le x\le\min(D,n)$)

Multinomial $(n;p_1,\ldots,p_m)$ $\left[P_{\underline{x}} = \binom{n}{\underline{x}} \prod_{i=1}^{m} (p_i^{x_i}/x_i!)\right]$ $\left(\sum_{i=1}^{m} p_i = 1; \underline{x} = (x_1,\ldots,x_m); 0 \le x_i, \sum_{i=1}^{m} x_i = n\right)$

$N(\xi,\sigma^2)$ $\left[\text{pdf} = (\sigma\sqrt{2\pi})^{-1}\exp(-\tfrac{1}{2}(x-\xi)^2\sigma^{-2})\right]$ $(\sigma>0)$

Beta (α,β) $\left[\text{pdf} = \{B(\alpha,\beta)\}^{-1}x^{\alpha-1}(1-x)^{\beta-1}\right]$ $(\alpha,\beta>0;\ 0<x<1)$

Gamma (α) $\left[\text{pdf} = \{\Gamma(\alpha)\}^{-1}x^{\alpha-1}e^{-x}\right]$ $(\alpha>0,\ 0<x)$.

We use the finite difference operator Δ, with definition

$$\Delta f(x) = f(x+1) - f(x)$$

to denote forward differences. We also write

$$x^{(m)} = x(x-1)\ldots(x-m+1) \text{ for the } m^{th} \text{ (descending) factorial of } x \tag{4}$$

$$x^{[m]} = x(x+1)\ldots(x+m-1) \text{ for the } m^{th} \text{ ascending factorial of } x\ . \tag{5}$$

3. Weighted Distributions and Damage Models

A variety of distributions can be constructed from models representing various degrees and kinds of imperfection in sampling and recording data.

If sampling is not random, but the probability of inclusion in the sample depends on the value of the variable to be measured, the result is a weighted distribution. From an original distribution $\{P_x^\circ\}$ and a weighting function $w(x)$ (= probability of inclusion in sample, given $X = x$) we obtain the weighted distribution

$$P_x = \frac{w_x P_x^\circ}{\sum_j w_j P_j^\circ}\ . \tag{6}$$

(Similar definitions are available for nondiscrete distributions.)

Since any discrete distribution (on $x = 0,1,\ldots$) can be derived in this way from any other (on $x = 0,1,\ldots$) with the same support, interest resides in particular, simple cases. Several practical examples are presented by Patil and Rao

(1977, 1978), including the "ascertainment problem" which has been the subject of study, for some time, by geneticists and epidemiologists (e.g., Fisher (1934)).

In the simple case when a distribution $\{P_x\}$ is modified by increasing P_{x_0} and compensating for this by multiplying the remaining P_x's by an appropriate constant, the resultant distribution - $\{P_x^*\}$ say - is called an *inflated* $\{P_x\}$ distribution. We have

$$\left.\begin{aligned} P_{x_0}^* &= (1-\omega) + \omega P_{x_0} \\ P_x^* &= \omega P_x \quad (x \ne x_0) \end{aligned}\right\} \tag{7}$$

with $0 < \omega < 1$. The pgf is $G^*(s) = (1-\omega)s^{x_0} + \omega G(s)$. If $\omega > 1$, the result is called a *deflated* distribution. The greatest possible value of ω is $(1-P_{x_0})^{-1}$. If ω takes this value, $P_{x_0}^* = 0$.

If $x_0 = 0$ (the case most commonly considered) and $P_0^* = 0$, then we have a *zero-truncated* - or, more picturesquely, "decapitated" - $\{P_x\}$ distribution. Yet another name is "positive $\{P_x\}$" distribution.

If $x_0 = 0$ and $0 < \omega < 1$ the phrase "with added zeroes" is sometimes used. General relations among the moments of $\{P_x\}$ and $\{P_x^*\}$ include:

$$\mu_r'^* = \omega\mu_r' + (1-\omega)x_0^r \tag{8.1}$$

$$\mu_{(r)}^* = \omega\mu_{(r)} + (1-\omega)x_0^{(r)}\ . \tag{8.2}$$

If $x_0 = 0$

$$\mu_r'^*/\mu_r' = \mu_{(r)}^*/\mu_{(r)} = \omega \text{ for all } r\ .$$

If there is a recurrence relation $g(\mu_1',\ldots,\mu_r') = 0$ for $\{P_x\}$, then $g\left(\frac{\mu_1'^*-(1-\omega)x_0}{\omega},\ldots,\frac{\mu_r'^*-(1-\omega)x_0^r}{\omega}\right) = 0$ for $\{P_x^*\}$. If $x_0 = 0$ we have

$g(n^{-1}\mu_1'^*,\ldots,n^{-1}\mu_r'^*) = 0$. If the relation is a linear one, $\sum_{j=1}^{r} c_j\mu_j' = c_0$, then we have $\sum_{j=1}^{r} c_j\mu_j'^* = c_0 n + (1-n)\sum_{j=1}^{r} c_j\, n_0^j$. (Grzegórska (1973a,b))

Another type of imperfection arises when observation is not always accurate. An example is the "faulty inspection" situation (Johnson *et al.* (1980)) wherein the chance of detecting that an item is defective is not 1, even when that item is selected in the sample. If the probability of detection is p, and the distribution of the actual number of defectives in the sample is F then the observed distribution is the compound

$$\text{Binomial}(Y,p) \underset{Y}{\wedge} F .$$

(For sampling without replacement from a lot of size N containing D defectives, F would be Hypg (n,D,N) where n is the sample size.)

These models belong to the general class of "damage models", in which the observed value X is not, in general, the original value Y, but what is left after some damage, resulting in reduction of Y to X, has been experienced, so that

$$X + Z = Y$$

where Z is the amount removed by damage. For example, in faulty inspection, Y is the actual number of defectives in a sample and X the number detected. The "damage" is incurred by the imperfection of the inspection.

Damage models have been studied extensively, mainly with a view to obtaining conditions characterizing the distribution of Y from that of X. This work originated with Rao and Rubin (JK, page 94) and has been progressively extended, and in some cases, proofs simplified by Govindarajulu and Leslie (1970), Talwalker (1970, 1980), Steele (1973), Shanbhag (1974, 1977), Krishnaji (1974), Aczel (1972, 1975), Consul (1974), Srivastava and Singh (1974), Patil and Ratnaparkhi (1975), Panaretos (1980), and Shanbhag and Taillie (1980).

The earlier papers were mostly concerned with results of the form: assuming X and Z to be mutually independent and the conditional distribution of X, given (X+Z), is

(i) *binomial*, then X and Z are each *Poisson*

(ii) *hypergeometric*, then X and Z are each *negative binomial*

(iii) *Pólya*, then X and Z are each *hypergeometric*.

Consul (1974b) and Janardan (1974) have obtained results of similar type for the (Lagrangian) double Poisson, quasi-binomial quasi-hypergeometric and quasi-Pólya (see Section 14) distributions. Multivariate extensions have been obtained by Shanbhag (1974a) and Panaretos (1980).

4. Negative Moments

Chao and Strawderman (1972) showed that, if r is a positive integer

$$E[(X+a)^{-r}] = \int_0^1 g_r(s)\,ds \qquad (9)$$

where $g_r(s)$ can be calculated from

$$\left.\begin{aligned} g_1(s) &= E[s^{X+a-1}] = s^{a-1}G(s) \\ \text{and}\quad g_{h+1}(s) &= s^{-1}\int_0^s g_h(t)\,dt \quad (h=1,2,\ldots r-1) \end{aligned}\right\}\ a \neq 0 \qquad (10)$$

Kabe (1976), generalizing Stancu's (1968) approach, obtained the formula

$$E[(X+a)^{-r}] = \sum_j \frac{\Delta^j a^{-r}}{j!}\mu_{(j)}, \quad a \neq 0. \qquad (11)$$

In particular cases, special techniques may be used. For example, Gupta (1979) uses a recurrence relation to obtain the expected value of the reciprocal of a zero-truncated generalized (Lagrange) double Poisson variable. See Kumar and Consul (1979) on modified power series with applications to generalized

(Lagrange) double Poisson and binomial distributions. See also, Law (1976) for some general bounds on inverse moments.

5. Minimum Variance Unbiased Estimators (MVUE)

A considerable amount of work has been done in the period on the construction of MVUE's, but little attention has been paid to possible usefulness of these estimators in particular cases. In many cases the sum (n-fold convolution) $Z_n = \sum_{j=1}^{n} X_j$ of n independent variables each having the distribution F is a sufficient statistic for the parameter (θ, say) being estimated, and so the MVUE is a function of Z_n. As a consequence the distribution of Z_n is of importance. Particular attention has been devoted to determining convolutions of decapitated, and other truncated distributions (Ahuja (1971b); Ahuja and Enneking (1974); Saleh and Rahim (1972)).

If P_x is of the general linear exponential form, then

$$P_x = g_1(\theta)g_2(x)\exp(xg_3(\theta)) \tag{12}$$

$$\Pr[Z_n=z] = (g_1(\theta))^n S_n(z)\exp(zg_3(\theta)) \tag{13}$$

where $S_n(z) = \sum\cdots\sum_{\Sigma x_j = z} \prod_{j=1}^{n} g_2(x_j)$. From (13) it can be seen that Z_n is a sufficient statistic for θ.

If, now $\{P_n\}$ be truncated by changing $g_2(x)$ to zero for some x's and changing

$$g_1(\theta) = [\sum_{x=0}^{\infty} g_2(x)\exp(xg_3(\theta))]^{-1}$$

to $g_1^*(\theta)$, appropriately then Z_n is still sufficient for θ. If we have zero-truncation so that

$$\Pr[Z_n=z] = (g_1^*(\theta))^n S_n^*(z)\exp(zg_3(\theta)) \quad (z=n,n+1\ldots) \tag{14}$$

with $S_n^*(z)$ obtained from $S_n(z)$ by omitting all terms in the summation in which any x_j is zero, the essential technical problem in determining the convolution of the decapitated variable is the evaluation of this sum. If $S_n(z)$ (for the untruncated distribution) is known we can use the recurrence formula

$$S_n^*(z) = S_n(z) - ng_2(0)S_{n-1}^*(z) - \binom{n}{2}(g_2(0))^2 S_{n-2}^*(z) - \ldots - n(g_1(0))^{n-1}g_2(z) \tag{15}$$

to determine $S_n^*(z)$'s prgressively, starting from $S_1(z) - S_1^*(z) = g_2(z)$.

6. Infinite Divisibility

Warde and Katti (1971) show that the discrete distribution $\{P_x\}$ on $x = 0,1,\ldots$, with $P_0 \neq 0$ and $P_1 \neq 0$ is infinitely divisible if the sequence $\{P_{j+1}/P_j\}$ increases with j. This condition is, in fact, quite a strong one; it requires that $\{P_x\}$ be a decreasing sequence. However, it is satisfied by geometric and logseries distributions, *inter alia*. Warde and Katti also give a method for constructing a new infinitely divisible distribution from a known one.

James (1976) shows that if $P_0 \neq 0$ then a necessary condition for infinite divisibility is

$$P_0 \operatorname{var}(X) \geq P_1 \,. \tag{16}$$

If this is an equality, the distribution must be Poisson. (See also Schmetterer (1970).)

Bondesson (1978, 1979) has introduced the idea of a *generalised negative binomial convolution* (g.n.b.c.). "Generalised" applies to "convolution" and not to "negative binomial". A g.n.b.c. is any distribution which can be obtained as a "weak limit of finite convolutions of negative binomial distributions". He demonstrates the parallelism of g.n.b.c.'s with "generalised gamma convolutions" defined by Thorin (1978). Among distributions identified as being g.n.b.c.'s, and so infinitely divisible, is

$$P_x = \frac{\beta^{[x]}}{x!} \prod_{j=1}^{r} (x+\alpha_j+\beta-1)^{-\gamma_j} \tag{17}$$

where $\alpha_j, \gamma_j > 0$ and $1 \le \beta \le \sum_{j=1}^{r} \gamma_j$.

7. <u>A General Result</u>

Rossmann and Warmuth (1979) have shown that

$$P_0 \ge \{(2+\rho)\mu_1' - \mu_2'\}\rho^{-1} (\rho+1)^{-1} \tag{18}$$

where ρ is the integer part of μ_2'/μ_1', for *any* discrete non-negative unit lattice distribution.

8. <u>Systems of Distributions</u>. <u>Power Series and Factorial Series Distributions</u>

Among discrete distributions, the class of power series distributions (Patil (1962, 1965)) is one of the most useful, in the sense described in Section 1.1. They are defined by

$$P_x = \frac{\theta^x}{x!} \cdot \frac{b(x)}{g(\theta)} \quad (\theta > 0;\ x = 0,1,\ldots) . \tag{19}$$

$g(\theta)$ and θ are called the *generating function* and the *parameter*, respectively,

of the distribution. Since $\sum_{x=0}^{\infty} P_x = 1$

$$g(\theta) = \sum_{x=0}^{\infty} \frac{\theta^x}{x!} b(x) \tag{20}$$

and so, subject to some regularity conditions

$$b(x) = D^x g(\theta)|_{\theta=0}. \tag{21}$$

If X has distribution (19), we write

$$X \frown PSD(\theta; g(\theta)) .$$

Modified power series distributions (Gupta (1974b)) are distributions with θ^x in (19) replaced by $\{u(\theta)\}^x$. Given $g(\theta)$, values of $b(x)$ can be obtained, if a Lagrange expansion (see (57)) can be used, from the formula

$$b(x) = D^{x-1}\{(\frac{\theta}{u(\theta)})^x g'(\theta)\}_{\theta=0} . \tag{22}$$

See also R. C. Gupta (1974b) and Kumar and Consul (1979) (Section 4) for further developments and applications. Janardan (1980b) discusses a "discrete exponential" family:

$$P_x = h(x) \exp\{\int (x - C(\theta)) B(\theta) d\theta\} \tag{23}$$

with $B(\theta)$, $C(\theta)$ positive and differentiable, and $C(\theta)$, $C'(\theta)$ strictly increasing; $h(x) \ge 0$.

A related class (see Irwin (1975), Berg (1974, 1975a,b, 1979, 1980)) is that of *factorial series distributions*. The factorial series distribution with generating function $h(N)$ and parameter N (an integer) is defined by

$$P_x = \frac{N^{(x)}}{x!} \frac{c(x)}{h(N)} \quad (x = 0,1,\ldots,N) \tag{24}$$

with

$$h(N) = \sum_{x=0}^{N} \frac{N^{(x)}}{x!} c(x) \tag{25}$$

so that

$$c(x) = \Delta^x h(N)\big|_{N=0} = \Delta^x h(0). \tag{26}$$

We write

$$X \sim FSD(N;h(N)) .$$

There are close similarities between properties of PSD's and FSD's. For example, the r^{th} factorial moment is

$$\mu_{(r),P} = \{\theta^r D^r g(\theta)\}/g(\theta) \tag{27}$$

for the PSD (19), and

$$\mu_{(r),F} = \{N^{(r)}\Delta^r h(N-r)\}/h(N) \tag{28}$$

for the FSD (24).

The distribution of the sum of n independent random variables $X_1, X_2, \ldots, X_n$ each having the PSD (19) is also a PSD, with generating function $\{g(\theta)\}^n$. Berg (1979) obtains an analogous result for FSD's which, however, requires the X's to have a certain kind of dependence. He introduces a "parameter translated" modification of FSD's, defined by

$$\Pr[X=x] = \frac{(N-a)^{(x)}}{x!}\Delta^x h(0+a)/h(N) \quad (x=0,1,\ldots,N-a) \tag{29}$$

written

$$X \sim PSD(N-a;h(N))$$

where a is a positive integer. Then if

$$X_1 \sim PSD(N;h(N));\ X_2 | X_1 \sim PSD(N-X_1;h(N))\ldots;$$

and generally

$$X_{j+1} | X_j \sim PSD(N - \sum_{i=1}^{j} X_i;h(N)),$$

it follows that

$$X_1+\ldots+X_n \sim FSD(N;\{h(N)\}^n) .$$

Berg (1979) also studies compound distributions of form

$$FSD(N;\{h(N)\}^n) \bigwedge_n PSD(\theta;g(\theta)) \tag{30}$$

and (Berg (1978b, 1980)) applies them in a generalized form of "snowball sampling". (Goodman (1961)).

Multivariate power series distributions (Patil (1965), JK, page 33) are defined by

$$P_{\underline{x}} = \prod_{i=1}^{m}\left\{\frac{\theta_i^{x_i}}{x_i!}\right\} b(\underline{x})/g(\underline{\theta}) \tag{31}$$

with

$$b(\underline{x}) = \left(\prod_{i=1}^{m} D_i^{x_i}\right)g(\underline{0})) \tag{32}$$

where D_i denotes differentiation with respect to θ_i. We write

$$\underline{X} \sim M_m PSD(\underline{\theta},g(\underline{\theta})).$$

Joshi and Patil (1971, 1974) have studied a special class of M_m PSD, introduced by Patil (1968), called *sum-symmetric* ("SSM_m PSD"), for which $g(\underline{\theta})$ is a function only of $\theta = \sum_{i=1}^{m}\theta_i$.

Multivariate *factorial series distributions* are defined by Berg (1977) analogously to M_m PSD, with

$$P_{\underline{x}} = \left\{\prod_{i=1}^{m}\frac{N_i^{(x_i)}}{x_i!}\right\} c(\underline{x})/h(\underline{N}) \tag{33}$$

where

$$c(\underline{x}) = \left(\prod_{i=1}^{m}\Delta_i^{x_i}\right)h(\underline{0}) . \tag{34}$$

(Δ_i denotes forward differencing with respect to N_i.) We write

$$\underline{X} \sim M_m FSD(\underline{N}, h(\underline{N})) .$$

The pgf is

$$G_{\underline{X}}(\underline{s}) = \left\{ \prod_{i=1}^{m} (1+s_i\Delta_i)^{N_i} h(\underline{0}) \right\} / h(\underline{N}) . \tag{35}$$

Sum-symmetric M_mFSD's (SSM_mFSD) are defined as those for which $h(\underline{N})$ is a function only of $N = \sum_{i=1}^{m} N_i$. Their properties are analogous to those of SSM_mPSD's, as indicated in the following table.

TABLE 1

SSM_mPSD (Joshi and Patil (1971, 1974))	SSM_mFSD (Berg (1977))
$X = \sum_{i=1}^{m} X_i \sim PSD(\theta, g(\theta))$	$X = \sum_{i=1}^{m} X_i \sim FSD\ (N, h(N))$
$\mu_{(\underline{r})}(\underline{X}) = \{\prod_{i=1}^{m} \theta_i^{r_i}\}\, \mu_r(X)/\theta^r$	$\mu_{(\underline{r})}(\underline{X}) = \{\prod_{i=1}^{m} N_i^{(r_i)}\}\, \mu_{(r)}(X)/N^{(r)}$

Next, we discuss a subclass of PSD's, with special features rendering them worthy of special attention.

9. Generalized Hypergeometric (Series) Distributions.

The generalized hypergeometric function

$${}_hF_k(\underline{a};\underline{b};\theta) = {}_hF_k(a_1,\ldots,a_h;b_1,\ldots,b_k;\theta) = \sum_{j=0}^{\infty} \left\{ \frac{\prod_{i=1}^{h} a_i^{[j]}}{\prod_{i=1}^{k} b_i^{[j]}} \right\} \frac{\theta^j}{j!} \tag{36}$$

can be used to construct a broad class of discrete distributions with

$$P_x = [{}_hF_k(\underline{a};\underline{b};\theta)]^{-1} \left\{ \frac{\prod_{i=1}^{h} a_i^{[x]}}{\prod_{i=1}^{k} b_i^{[x]}} \right\} \frac{\theta^x}{x!} \quad (x=0,1,\ldots) \tag{37}$$

provided no P_x's are negative and the series converges (Kemp (1968)). Note that $a^{[0]} = 1$ so that

$$P_0 = [{}_hF_k(\underline{a};\underline{b};\theta)]^{-1} . \tag{38}$$

The pgf is

$${}_hF_k(\underline{a};\underline{b};\theta s)/{}_hF_k(\underline{a};\underline{b};\theta) . \tag{39}$$

We may write

$$X \sim {}_hG\ Hyp_k(\underline{a};\underline{b};\theta) . \tag{40}$$

These *generalized hypergeometric (series) distributions* form a very extensive class and have some specially attractive features (such as simple recurrence relationships for probabilities, factorial moments, etc.). Dacey (1972), Table 2) gives a list of some 50 pgf's expressed in terms of ${}_pF_q$ functions, with $p,q \le 3$.

The formula for the ratio of successive probabilities

$$\frac{P_{x+1}}{P_x} = \frac{\theta}{x+1} \cdot \frac{\prod_{i=1}^{h}(a_i+x)}{\prod_{i=1}^{k}(b_i+x)} \tag{41}$$

may be regarded as a generalization of the formula

$$\frac{P_{x+1}}{P_x} = \frac{\alpha+\beta x}{1+x} \tag{42}$$

used by Katz (1963) to generate a family of distributions. (In the symbolism of (40) above, (42) corresponds to

$$X \sim {}_1G\ Hyp_0(\tfrac{\alpha}{\beta};-;\beta).)$$

The subclass ${}_2G\ Hyp_1(a_1,a_2;b_1;\theta)$ has been used much more than any other members of the class. In Dacey's (1972) list, about 30, among 50 entries use ${}_2F_1$. Kemp and Kemp (1968, 1969, 1971) discuss a number of interesting special cases, including "lost-games" distribution - that of the number of games lost by a player starting with $\underline{a}$ units of money before losing all his money, when in each game the probability of losing a unit is p (and of gaining a unit, q), and (1969) distributions having applications in epidemic theory and queueing theory.

They also show (1971) that these distributions can be obtained in a variety of ways as mixtures of negative binomial or Poisson distributions.

More recently Kemp and Kemp (1974) introduced a class of distributions with pgf ${}_pF_g((a);(b);\lambda(z-1))$ which they called *generalized hypergeometric factorial-moment distributions*. Besides some well-known distributions like Poisson, negative binomial and Pólya, this class contains some matching and occupancy distributions (see Johnson and Kotz (1977) and Kemp and Kemp (1978)) and other compound distributions. Again, recurrence relationships for probabilities, moments, etc., are easily obtainable.

Gurland and Tripathi (1974, 1977) consider the more general class - termed by them "extended Katz" (EK) for which

$$\frac{P_{x+1}}{P_x} = \frac{\alpha+\beta x}{\lambda+x}\ (\alpha,\ \lambda>0;\ 0<\beta<1) \tag{43}$$

which has $h = 2$, $k = 1$, $a_1 = \alpha/\beta$, $a_2 = 1$, $b_1 = \lambda$, $\theta = \beta$. The factorial moments satisfy the recurrence relation

$$(1-\beta)\mu_{(r+2)} + \{\lambda+r-\alpha-2(r+1)\beta\}\mu_{(r+1)} - (r+1)(\alpha+r\beta)\mu_{(r)} = 0. \tag{44}$$

Also

$$E[X] = \frac{\alpha - (\lambda-1)(1-P_0)}{1-\beta}. \tag{45}$$

The class is extended to negative values of β by using the function

$${}_2F_1(\alpha/\beta,\ 1;\lambda;\beta) = \sum_{j=0}^{N} \frac{(\alpha/\beta)^{[j]}}{\lambda^{[j]}}\frac{\beta^j}{j!} \tag{46}$$

with N equal to the integer part of $-\alpha/\beta$. As $\beta\to0$ (either through positive or negative values) the class (CB) described by Crow and Bardwell (1963) - and termed

hyper-Poisson by these authors - is approached. The pgf for this class is

$${}_1F_1(1;\lambda;\alpha s)/{}_1F_1(1;\lambda;\alpha)\ . \tag{47}$$

Gurland and Tripathi (1974) extend the CB class to

(i) E_1 CB, with $P_{x+1}/P_x = \alpha(\gamma+x)(\lambda+x)^{-1}(1+x)^{-1}$ (48)

and pgf: ${}_1F_1(\gamma;\lambda;\alpha s)/{}_1F_1(\gamma;\lambda;\alpha)$ (49)

(with *either* $\alpha,\gamma,\lambda > 0$ *or* $\alpha > 0$ and $\gamma,\lambda < 0$ with γ and λ having the same integer parts) and

(ii) E_2 CB, with pgf ${}_1F_1(\gamma;\lambda;\alpha_0+\alpha_1 s)/{}_1F_1(\gamma;\lambda;\alpha_0+\alpha_1)$. (50)

This class (ii) is *not* included in the general form (37). It *includes* (i) as a special case with $\alpha_0 = 0$. Conditions on the parameters are

$\alpha_0,\ \alpha_1,\ \gamma,\ \lambda > 0$

or

$\lambda > \gamma > 0,\ \alpha_1 > 0$

or

$\gamma < 0$, $\lambda < 0$, γ and λ having the same integer parts, $\alpha_1 > 0$, $\alpha_0+\alpha_1 \geq 0$.

Ord (1972), following K. Pearson's (1895) original approach, obtains a system of hypergeometric (series) distributions as solutions of the difference equation

$$P_x - P_{x-1} = \frac{(a-x)P_{x-1}}{b_0+b_1x+b_2x^{(2)}}\ . \tag{51}$$

Ord (1972, Table 5.1) gives a summary of the members of this system. Note that for all cases except Type IX, $b_0 = 0$. As a way of distinguishing among those different members, for which $b_0 = 0$, Ord notes that (51) can then be rewritten

$$\frac{xP_x}{P_{x-1}} = x + \frac{a-x}{b_1+b_2(x-1)}\ . \tag{52}$$

If $b_2 = 0$, xP_x/P_{x-1} is a linear function of x.

It is suggested that $u_x = xf_x/f_{x-1}$ be plotted against x, and if the plot appears to be linear, the following table (based on Table 5.4 of Ord (1972)) may be used to choose the appropriate distribution.

Intercept ("u_0")	Slope	Distribution
$\phi > 0$	ϕ	Poisson (ϕ)
$(n+1)p/q > 0$	$-p/q < 0$	Binomial (n,p)
$(N-1)P/Q > 0$	$P/Q > 0$	Negative binomial (N,P)
$-\phi < 0$	$\phi > 0$	Log series (ϕ) (note: intercept = -(slope))
0	1	Discrete rectangular

When $\theta = 1$ we have *generalised hypergeometric* distributions ("(series)" is dropped).

A considerable part of the increased attention devoted to multivariate discrete distributions in recent years has been directed to generalizations of multivariate hypergeometric distributions. (Note that "generalized" is not used in the limited "random sum" sense.)

Although this work has been done fairly recently, most of the distributions discussed are included in a very broad class described by Steyn (1951) some 30 years ago. For this class

$$P_{\underline{x}} = \frac{(c - \sum_{j=1}^{m} b_j - a)^{[a]}}{c^{[a]}} \cdot \frac{a^{[\sum x_j]}}{c^{[\sum x_j]}} \prod_{j=1}^{m} \frac{b_j^{[x_j]}}{x_j!} \qquad (53)$$

the values of a, b_j and c being such that $P_{\underline{x}} \geq 0$.

Janardan and Patil (1972) show that this class includes many established distributions, among them the multivariate hypergeometric (JK, page 300), inverse hypergeometric, negative hypergeometric, negative inverse hypergeometric, Pólya, and inverse Pólya distributions. They also summarize some possible geneses of these distributions.

10. Digamma and Trigamma Distributions.

Sibuya (1979a), by taking limiting cases of the zero-truncated generalized hypergeometric (inverse Pólya-Eggenberger) ({Negative Binomial $(\alpha,P) \underset{P/\alpha}{\wedge}$ Beta (β,γ)}) distribution

$$P_x = \frac{\gamma^{[\alpha]}\gamma^{[\beta]}}{\gamma^{[\alpha+\beta]}} \cdot \frac{\alpha^{[x]}\beta^{[x]}}{x!(\alpha+\beta+\gamma)^{[x]}} \cdot \left(1 - \frac{\gamma^{[\alpha]}\gamma^{[\beta]}}{\gamma^{[\alpha+\beta]}}\right)^{-1} \quad (x = 1,2,\ldots) \qquad (54)$$

(interpreting $a^{[b]}$ as $\Gamma(a+b)/\Gamma(a)$)
constructed two new distributions as limiting cases.

(i) As $\beta \to 0$ (with $\alpha \neq 0$)

$$P_x \to \{\psi(\alpha+\gamma)-\psi(\gamma)\}^{-1} \frac{\alpha^{[x]}}{(\alpha+\gamma)^{[x]}x} \quad (x=1,2,\ldots;\ \gamma>0;\ \alpha>-1;\ \alpha+\gamma>0) \qquad (55)$$

where $\psi(y) = d \log \Gamma(y)/dy$ is the digamma function. This is termed a *digamma distribution*.

(ii) As $\alpha \to 0$ and $\beta \to 0$

$$P_x \to \frac{1}{\psi'(\gamma)} \cdot \frac{(x-1)!}{\gamma^{[x]}x} \qquad (x = 1,2,\ldots;\ \gamma > 0). \qquad (56)$$

The function $\psi'(y) = d\psi(y)/dy$ is the trigamma function, and (56) is termed a *trigamma distribution.*

Clearly the trigamma distributions are limit distributions (as $\alpha \to 0$) of the digamma distributions. If α and γ tend to infinity with $\alpha(\alpha+\gamma)^{-1} = \theta$ the distribution tends to a logseries distribution with parameter θ.

The trigamma distribution (56) is very similar to the zeta distribution (JK page 240). In fact, when $\gamma = 1$, the trigamma distribution *is* a zeta distribution with parameter 1. Sibuya (1979b) suggests that the digamma distribution may be used in place of logseries distributions when the tails of the latter are not long enough to fit the data. Multivariate digamma distributions can be derived from a truncated multivariate inverse Polya-Eggenberger, but when a multivariate trigamma is sought, it turns out to be degenerate.

11. Lagrange Distributions.

Exploitation of the Lagrange series expansion

$$f(s) = f(0) + \sum_{j=1}^{\infty} \frac{u^j}{j!} D^{j-1}\{[g(t)]^j f'(t)\}|_{t=0} \qquad (57)$$

where $u = s/g(s)$ has produced some useful distributions. There is a good general introduction in Consul and Shenton (1972b), and useful information on construction of Lagrange distributions in Jain (1974a).

The functions $f(\cdot)$ and $g(\cdot)$ are each taken to be pgf's. Since $s = 1$ when $u = 1$, $s = s(u)$ can also be a pgf, and will be so if the coefficients of powers of u in the expansion of $s(u)$ are all positive. Then (57) will represent the pgf with argument u of a "generalization" (in the sense of Gurland - see Chapter 8 of JK) of the distribution with pgf $f(\cdot)$ by the distribution with pgf $g(\cdot)$. For these distributions

$$\left.\begin{aligned} P_0 &= f(0) \\ P_x &= D^{x-1}\{[g(t)]^x f'(t)\}|_{t=0}/x! \; (x = 1,2,\ldots) \, . \end{aligned}\right\} \qquad (58)$$

They form the class of *Lagrange distributions*. We write

$$X \sim L(g(\cdot);\, f(\cdot)) \, . \qquad (59)$$

Specific subclasses are obtained by choosing specific forms for $g(\cdot)$ and $f(\cdot)$ termed, respectively, the *transformation and generating* pgf's for $L(g(\cdot);\, f(\cdot))$.

For example, the Poisson-Poisson (Lagrange "double Poisson" distribution, or generalized Borel-Tanner distribution (JK, page 254)) is obtained by taking $g(s) = \exp(\lambda_2(s-1))$; $f(s) = \exp(\lambda_1(s-1))$ - the pgf's of Poisson distributions with parameters λ_2, λ_1 respectively. In order to satisfy the condition on $s(u)$ (see above) we must have $\lambda_2 < 1$. For this distribution

$$P_x = \lambda_1(\lambda_1+\lambda_2 x)^{x-1} e^{-(\lambda_1+\lambda_2 x)}/x! \quad (x = 0,1,\ldots) \qquad (60)$$

(Consul and Jain (1970)).

Table 6.1 of Consul and Shenton (1972) lists thirteen Lagrange distributions obtained by combining binomial, Poisson, negative binomial and "delta" pgf's (the latter has $P_n = 1$). The same authors also describe Lagrange Poisson-rectangular and Poisson-logseries distributions.

Consul and Shenton (1972) show that

$$\left.\begin{aligned} E[X|L(g(\cdot);\, f(\cdot)] &= f_1(1-g_1)^{-1} \\ \mathrm{var}(X|L(g(\cdot);\, f(\cdot)) &= f_2(1-g_1)^{-2} + f_1 g_2(1-g_1)^{-3} \end{aligned}\right\} \qquad (61)$$

(where f_i, g_i are the i^{th} cumulants of the distributions with pgf's $f(s)$, $g(s)$ respectively and also give formulas for $\mu_3(X)$ and $\mu_4(X)$. If $g(\cdot)$ is such that $|g_1| < 1$, then all moments of $L(g(\cdot);\, f(\cdot))$ exist.

Consul and Shenton (1974) give a useful summary of properties of Lagrange distributions. They also identify several Lagrange distributions arising in queueing theory.

Consul and Shenton (1973) have studied certain limit cases of Lagrange distributions. They show that

(i) if g_1 is kept fixed and $f_1 \to \infty$ the standardized $L(g(\cdot);\, f(\cdot))$ distribution tends to normality, but

(ii) if $f_1 \to \infty$ and $g_1 \to 1$ in such a way that $f_1(1-g_1)g_2^{-1} \to c^2$, then the standardized distribution tends to the inverse Gaussian distribution with pdf

$$c^{3/2}(2\pi)^{-1/2}(y+c)^{-3/2}\exp(-\tfrac{1}{2}cy^2(y+c)^{-1}). \qquad (62)$$

Consul and Shenton (1972a), utilizing a multivariate extension of Lagrange's expansion due to Good (1959), described a general approach to construction of multivariate distributions with (univariate) Lagrange marginals. They showed that such distributions appeared naturally in queueing theory, gave general formulae for variances and covariances and constructed a multivariate Borel-Tanner ("double Poisson") distribution with

$$P_x = \left[\prod_{j=1}^{m} \frac{(\sum_{i=1}^{m}\lambda_{ij}x_i)^{x_j-r_j}\exp(-(\sum_i\lambda_{ji})x_j)}{(x_j-r_j)!}\right] |I-A(x)| \qquad (63)$$

where the (g,h)-th element of the $m\times m$ matrix $A(x)$ is $\frac{\lambda_{gh}(x_g-r_g)}{\sum_i \lambda_{ih}x_i}$, and the λ's are positive constants for $x_j \geq r_j$. A queueing theory interpretation is given.

The same authors (Shenton and Consul (1973)) consider the bivariate case in more detail.

Jain (1974a) extended his treatment of univariate Lagrange power series distributions to multivariate distributions. In particular Jain and Singh (1975) obtained the Lagrangian bivariate negative binomial

$$P_{x_1,x_2} = \frac{N(\Gamma(N+\gamma_1x_1+\gamma_2x_2))}{\Gamma(N+(\gamma_1-1)x_1+(\gamma_2-1)x_2+1)} \frac{(Q-P_1-P_2)^{N+(\gamma_1-1)x_1+(\gamma_2-1)x_2}}{x_1!x_2!Q^{N+\gamma_1x_1+\gamma_2x_2}} P_1^{x_1}P_2^{x_2} \qquad (64)$$

where $0 < P_1,P_2$; $P_1+P_2 < Q$; $N > 0$, $\gamma_1,\gamma_2 > 1$.

They also obtain a bivariate Borel-Tanner distribution

$$P_{x_1,x_2} = \frac{\lambda(\lambda+\lambda_1x_1+\lambda_2x_2)^{x_1+x_2-1}\exp\{-(\lambda+\lambda_1x_1+\lambda_2x_2)(\theta_1+\theta_2)\}}{x_1!x_2!}\theta_1^{x_1}\theta_2^{x_2} \qquad (65)$$

where $\lambda,\lambda_1,\lambda_2,\theta_1,\theta_2 > 0$, which differs from (63).

12. Discrete (Linear) Exponential Distributions.

These are defined by

$$P_x = g(\theta_1,\ldots,\theta_k)b(x)\exp(\sum_{i=1}^{k}\theta_it_i(x)). \qquad (66)$$

Barndorff-Nielsen (1973) contains a thorough discussion of this family.

Estimation of the θ_j's from $X_1,X_2,\ldots,X_m$ when the X's are independent and

$$\Pr[X_j=x] = g(\theta_j)b(x)e^{\theta_jt(x)} \quad (j=1,\ldots,m) \qquad (67)$$

is discussed by Tsui (1979a) who obtains results on admissibility (with squared error loss function) similar to those of Stein for multinormal distributions. Methods of shrinking the MVUE's to improve estimation are also discussed.

Recent papers by Lauritzen (1975) and Janardan (1980a) discuss some further generalizations.

13. 'Abel' Distributions.

Consul (1974a) and Consul and Mittal (1975) have derived a number of distributions which are related to Abel's generalizations (e.g., Riordan (1979)) of the binomial expansion (see also Janardan (1974)). These include

Quasi-binomial distribution I.

$$P_x = \frac{\alpha}{(\alpha+\beta+n\tau)^n}\binom{n}{x}(\alpha+\tau x)^{x-1}(\beta+\tau(n-x))^{n-x} \quad (x=0,1,\ldots,n). \qquad (68.1)$$

Quasi-binomial distribution II.

$$P_x = \frac{\alpha\beta}{\alpha+\beta}\binom{n}{x}\frac{(\alpha+\tau x)^{x-1}\{\beta+\tau(n-x)\}^{n-x-1}}{(\alpha+\beta+n\tau)^{n-1}} \qquad (x=0,1,\ldots,n). \qquad (68.2)$$

Quasi-hypergeometric distribution.

$$P_x = \frac{\alpha\beta(\alpha+\beta+\tau n)}{(\alpha+\beta)(\alpha+\tau x)\{\beta+\tau(n-x)\}}\binom{\alpha+\tau x}{x}\binom{\beta+\tau(n-x)}{n-x}\Big/\binom{\alpha+\beta+\tau n}{n} \quad (x=1,\ldots,n). \quad (69)$$

Quasi-Pólya distribution.

$$P_x = \binom{n}{x}\frac{\alpha\{\alpha+(x-1)\tau\}^{[x-1]}\{\beta+(n-x)\tau\}^{[n-x]}}{(\alpha+\beta+n\tau)^{[n]}} \qquad (x=0,1,\ldots,n). \qquad (70)$$

In all formulae (68)-(70) the parameters α,β,τ can be any positive numbers. Proper distributions are also obtained with some other sets of values of the parameters.

Multivariate extensions ("quasi-multinomial" distributions), based on Hurwitz's extensions of Abel's identities are described in Consul and Mittal (1977).

14. Concluding Remark.

The preceding text is based on a considerably more extensive (though still not exhaustive) original version, which the authors plan to publish, if opportunity arises.

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In this paper we survey and summarise developments in the theory and methodology of discrete distributions during the period 1969-1980 since publication of our book Distributions in Statistics - Discrete Distributions (Wiley 1969). A comprehensive (though not exhaustive) bibliography of some 400 items is included.

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